

Math 131A-1: Homework 4

Due: January 30, 2015

1. Read Sections 10-11 in Ross.
2. Do problems 9.10, 9.12, 9.14, 10.6, 10.7, and 10.10 in Ross. [You may assume the result of 9.13 in writing up 9.14, but understanding this exercise on your own time is also recommended.]
3. We say a subset S of \mathbb{R} is *closed* if whenever a sequence (s_n) of numbers in S converges to some s , the limit s is also in S .
 - (a) Use exercise (8.9) from last week to show that the interval $[a, b]$ is closed for any real $a < b$.
 - (b) Give an example of a closed unbounded set in \mathbb{R} (other than \mathbb{R} itself).
 - (c) Suppose $S \subset \mathbb{R}$ is closed and bounded above. Use exercise (10.7) above to show that S has a maximum.
4. Let (s_n) be the sequence $s_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$.
 - (a) Show that s_n is increasing and bounded above by 2. [Hint: Multiply s_n by 2^n , then use an induction formula for $2^n + 2^{n-1} + \cdots + 2^2 + 2 + 1$ proved in the second lecture. Later in the course we will see a general formula for this sort of sum.]
 - (b) Show that $s_{n+1} = \frac{1}{2}s_n + 1$, and conclude that $\lim s_n = 2$.
 - (c) Let $t_n = 1 + 2 + 4 + \cdots + 2^n$. Observe that $t_{n+1} = 2t_n + 1$. Why can't we conclude that $\lim t_n = -1$?
Note that this addresses one of our motivational questions from the first lecture!
5. The sample midterm isn't technically homework, but don't forget it exists.

NB: For your planning purposes, note that 10.6 is the only question above that uses material which is not fair game for the midterm.